Consider the sequence defined recursively by $a_n = 6a_{n-2} - a_{n-1}$, $a_1 = -2$, $a_2 = 6$.

SCORE: _____ / 15 PTS

[a] Write the first 5 terms of the sequence as a list.

$a_3 = 6a_1 - a_2 = 6(-2) - 6 = -18$

$a_4 = 6a_2 - a_3 = 6(6) - {}^{-}18 = 54$

$a_5 = 6a_3 - a_4 = 6(-18) - 54 = -162$

$-2, 6, -18, 54, -162$

[b] Based on the first 5 terms, is the sequence arithmetic, geometric or neither ? Justify how you arrived at your conclusion.

$\frac{6}{-2} = \frac{-18}{6} = \frac{54}{-18} = \frac{-162}{54} = -3$

GEOMETRIC

Find parametric equations for the following graphs.

SCORE: _____ / 20 PTS

[a] the line through $(-4, -3)$ and $(2, -8)$

$x = -4 + (2 - {}^{-}4)t$ $\quad$ $x = -4 + 6t$

$y = -3 + (-8 - 3)t$ $\quad$ $y = -3 - 5t$

[b] the portion of the graph $y = x^2 - 3x$ from $(-2, 10)$ to $(3, 0)$

$x = t$

$y = t^2 - 3t$

$t \in [-2, 3]$

Find the sum of the first 18 terms of the series $81-54+36-24+16-\cdots$. **SCORE: _____ / 10 PTS**

Round your final answer to 3 decimal places.

GEOMETRIC $r=\frac{-54}{81}=-\frac{2}{3}$

$$S_{18}=\frac{81\left(1-\left(-\frac{2}{3}\right)^{18}\right)}{1-\frac{-2}{3}}\approx 48.567$$

Find a rectangular equation corresponding to the parametric equations $\begin{array}{l} x=\frac{1}{2}\ln 3t \\ y=6t^2 \end{array}$. **SCORE: _____ / 15 PTS**

Write your final answer in the form y as a simplified function of x.

$2x=\ln 3t$

$e^{2x}=3t$

$t=\frac{1}{3}e^{2x}$

$y=6\left(\frac{1}{3}e^{2x}\right)^2$

$y=\frac{2}{3}e^{4x}$

Find the coefficient of $r^{48}t^{12}$ in the expansion of $(2r^6-3t^4)^{11}$. **SCORE: _____ / 15 PTS**

$\binom{11}{k}(2r^6)^{11-k}(-3t^4)^k$

$6(11-k)=48$ AND $4k=12$

$11-k=8$

$k=3$

$=\frac{11!}{3!\,8!}(2r^6)^8(-3t^4)^3$

$=\frac{11\cdot \cancel{10}^{5}\cdot \cancel{9}^{3}}{\cancel{3}\cdot\cancel{2}\cdot 1}(256)(-27)r^{48}t^{12}$

$=-1{,}140{,}480\,r^{48}t^{12}$

Using mathematical induction, prove that $\sum_{i=1}^{n} \frac{2}{(3n+2)(3n-1)} = \frac{n}{3n+2}$ for all positive integers n. **SCORE: _____ / 25 PTS**

BASIS STEP: $\sum_{i=1}^{1} \frac{2}{(3i+2)(3i-1)} = \frac{2}{5 \cdot 2} = \frac{1}{5} = \frac{1}{3 \cdot 1+2}$

INDUCTIVE STEP: ASSUME $\sum_{i=1}^{k} \frac{2}{(3i+2)(3i-1)} = \frac{k}{3k+2}$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

$$\sum_{i=1}^{k+1} \frac{2}{(3i+2)(3i-1)} = \sum_{i=1}^{k} \frac{2}{(3i+2)(3i-1)} + \frac{2}{(3k+5)(3k+2)}$$

$$= \frac{k}{3k+2} + \frac{2}{(3k+5)(3k+2)}$$

$$= \frac{k(3k+5)+2}{(3k+5)(3k+2)}$$

$$= \frac{3k^2+5k+2}{(3k+5)(3k+2)}$$

$$= \frac{(3k+2)(k+1)}{(3k+5)(3k+2)}$$

$$= \frac{k+1}{3k+5}$$

$$= \frac{(k+1)}{3(k+1)+2}$$

SO BY MI, $\sum_{i=1}^{n} \frac{2}{(3i+2)(3i-1)} = \frac{n}{3n+2}$ FOR ALL INTEGERS $n \geq 1$

Use sigma notation to write the series $\frac{32 \cdot 3}{2} + \frac{48 \cdot 11}{6} + \frac{72 \cdot 19}{24} + \frac{108 \cdot 27}{120} + \cdots + \frac{243 \cdot 43}{5040}$. **SCORE: _____ / 15 PTS**

NUMERATOR 1^{ST} FACTOR GEOMETRIC $r = \frac{3}{2}$

2^{ND} FACTOR ARITHMETIC $d = 8$

DENOMINATORS $= 2!, 3!, 4!, 5!, \ldots, 7!$

$$\sum_{n=1}^{6} \frac{32(\frac{3}{2})^{n-1}(3+8(n-1))}{(n+1)!} = \sum_{n=1}^{6} \frac{32(8n-5)3^{n-1}}{(n+1)!\, 2^{n-1}}$$

Write $0.2\overline{45}$ as a simplified fraction using the techniques of chapter 9. **NOTE: Only the 45 is repeated.** **SCORE: _____ / 15 PTS**

$0.2 + 0.045 + 0.00045 + 0.0000045 + \cdots$

$= \frac{1}{5} + \frac{\frac{45}{1000}}{1 - \frac{1}{100}}$

$= \frac{1}{5} + \frac{\cancel{45}^{\,9}}{\cancel{1000}_{\,2}} \cdot \frac{\cancel{100}}{\cancel{99}_{\,11}}$

$= \frac{1}{5} + \frac{1}{22}$

$= \frac{27}{110}$

Pat started saving for retirement by making monthly deposits into a savings account. Pat's strategy was to start with a small first deposit, and then increase the monthly deposit by a fixed dollar amount each month. If the sixth deposit was $312 and the tenth deposit was $385, how much was deposited during the first 3 years? **SCORE: _____ / 20 PTS**

$a_6 = a_1 + 5d$

$a_{10} = a_1 + 9d$

$312 = a_1 + 5d$

$385 = a_1 + 9d$ SUBTRACT

$73 = 4d$

$d = 18.25$

$312 = a_1 + 5(18.25)$

$a_1 = 220.75$

$S_{36} = \frac{36}{2}(2(220.75) + 35(18.25))$

$= \$19{,}444.50$